

Dom- Chromatic Number on Various Classes of Graphs

S. Anusha

*Assistant professor, Department of Mathematics, S. T. Hindu College, Nagercoil-629002,
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012,
Tamilnadu, India.*

Abstract

Let G be a graph. For a given χ - colouring of a graph G , a dominating set $S \subseteq V(G)$ is said to be a dom-colouring set if it contains atleast one vertex of each colour class of G . The dom chromatic number of a graph G is the minimal cardinality taken over all its dom-colouring sets and is denoted by $\gamma_{d-c}(G)$. In this paper, we introduce algorithms to obtain the dom-colouring and dom-chromatic number of various classes of graphs.

Keywords: Chromatic Number, Dominating Set, Domination Number, Dom-Colouring Set, Dom-Chromatic Number, Dom- chromatic number, central graph of cycle, central graph of jelly fish, tree and complete graph.

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1. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let $G = (V, E)$ be a graph. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. An induced subgraph $G[S]$, where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S . A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice -verse. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and $n-1$ leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H , we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$.

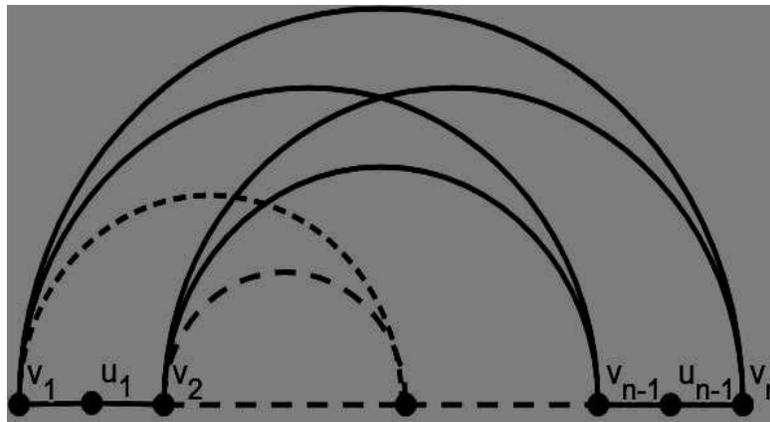
A subset S of V is called a dominating set if every vertex in $V-S$ is adjacent to atleast one vertex in S . The dominating set is minimal dominating set if no proper subset of S is a dominating set of G . The domination number γ is the minimum cardinality taken over all minimal dominating set of G . A γ -set is any minimal dominating set with cardinality γ .

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A dominator coloring of G is a proper coloring of G in which every vertex of G dominates atleast one color class. The dominator

chromatic number is denoted by $\chi_d(G)$ and is defined by the minimum number of colors needed in a dominator coloring of G . This concept was introduced by Ralucca Michelle Gera in 2006[2]. In a proper coloring C of G , a color class of C is a set consisting of all those vertices assigned the same color. Let C^l be a minimal dominator coloring of G . We say that a color class $c_i \in C^l$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G . These color classes are also called repeated color classes. For a given χ -colouring of a graph G , a dominating set $S \subseteq V(G)$ is said to be a dom-colouring set if it contains atleast one vertex of each colour class of G . The dom chromatic number of a graph G is the minimal cardinality taken over all its dom-colouring sets and is denoted by $\gamma_{d-c}(G)$.

Let G be an undirected graph with no loops and parallel edges. The graph formed by subdividing each edge exactly once and joining all the non-adjacent vertices of the graph is called the central graph. It is denoted by the symbol $C(G)$.

Example:



Main results

Theorem 1:

The dom- chromatic number for central graph of cycle of length n is 6. (ie) $\gamma_{d-c}(C(Cn)) = 6$

Proof:

Let the central graph is denoted by C_n .

The vertex set $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(C_n) = \{e_1, e_2, \dots, e_n\}$ where $e_i = v_i, v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n, v_1$. The central graph of length n is denoted by $C(C_n)$.

The central graph $C(C_n)$ is formed by subdividing each edge $e_i = v_i, v_{i+1}, 1 \leq i \leq n-1$ of C_n exactly once by adding a new vertex u_i and subdividing $e_n = v_n, v_1$, by u_n and joining v_i with $v_j, 1 \leq i, j \leq n, i \neq j$.

The new vertex set formed is $V(C(C_n)) = \{V_1 \cup U_1\}$ where $V_1 = \{v_1, v_2, \dots, v_n\}$ and $U_1 = \{u_1, u_2, \dots, u_n\}$. The new edge set formed is $E(C(C_n)) = \{E_1 \cup E_2\}$ such that $E_1 = \{e_1', e_2', \dots, e_n'\}$ where $e_k = v_i, v_j, 1 \leq i, j \leq n, k = 1, 2, 3, \dots, n$ and $i \neq j$ and $E_2 = \{e_1'', e_2'', \dots, e_n''\}$ where $e_i'' = u_i, v_{i+1}$.

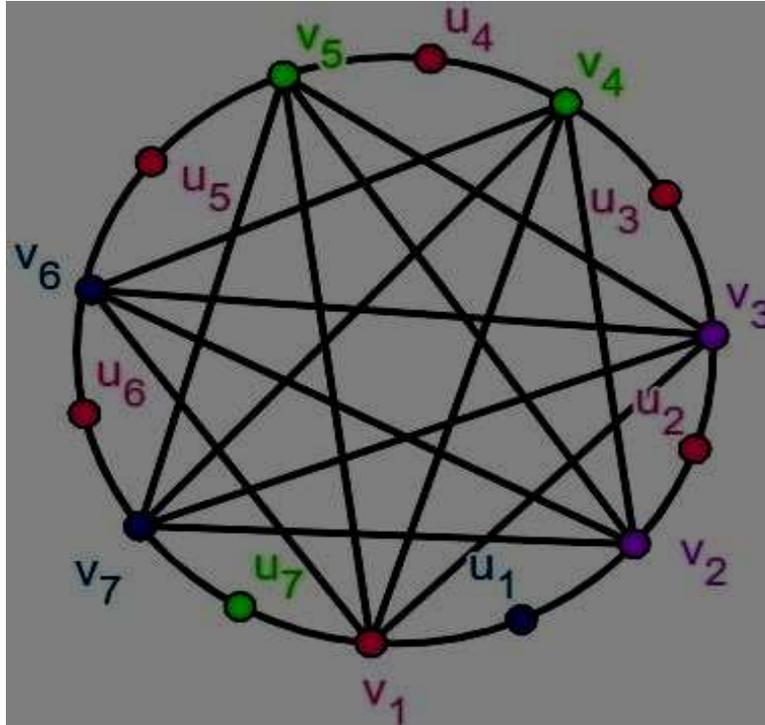
Define a mapping $\varphi: V(C(C_n)) \rightarrow \{1, 2, 3, 4\}$ such that $\varphi(v_1) = 1, \varphi(v_2) = \varphi(v_3) = 2, \varphi(v_4) = \varphi(v_5) = 3, \varphi(v_6) = \varphi(v_7) = 4$ and $\varphi(u_1) = 4, \varphi(u_2) = \varphi(u_3) = \varphi(u_4) = \varphi(u_5) = \varphi(u_6) = 1, \varphi(u_7) = 3$. Clearly it is a proper dom-coloring on vertices and $\gamma_{d-c}(\dots) \geq 4$. Suppose assume that $\varphi(\dots) \geq 4$.

= 5 by some optimal dom-coloring β . Then the dom-coloring β assigns distinct colors to higher degree non adjacent vertices. Therefore, colors on v_1, v_2, v_4, v_7 must be distinct.

Now, the fifth color must appear on any of the remaining vertices. If this happens, then there exist any one pair of color with non-adjacent vertices does not have edge between them which is contradiction. So we have $\chi((C_n)) \leq 4$. Therefore, from the inequalities we arrive the result. Hence the dom-chromatic number for central graph of cycle of length n is 2.

We have $\gamma d-(G) = \chi(G) + \gamma(G) - 1$. Then $\gamma d-(C(C_n)) = \chi(C(C_n)) + \gamma(C(C_n)) - 1$

Therefore, $((C_n)) = 4 + 3 - 1 = 6$.



Theorem 2:

The dom-chromatic number for central graph of jelly fish graph 4. (ie) $\gamma d-(C(J(m, n)))=4$.

Proof:

Let the jelly fish graph is denoted by (m, n) . The vertex set be $((m, n)) = \{x_1, x_2, x_3, x_4 \cup u_1, u_2, \dots, u_m \cup v_1, v_2, \dots, v_n\}$ and the edge set be $E(J(m, n)) = \{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_1),$

$(x_1, x_3) \cup (x_4, u_i) \cup (x_2, v_j)\}$ where $i = 1, 2, 3, \dots, n$ and $j = 1, 2, 3, \dots, m$.

The central graph of jelly fish graph is denotrd by the symbol $((m, n))$. The central graph of $((m, n))$ is formed by subdividing each edge $(x_i, x_{i+1}), 1 \leq i \leq n$ exactly once by adding a new vector c_i and joining x_i with $x_{i+2}, i = 1, 2, 3, 4$ and inclusion of vertex c_5 between the edge (x_1, x_3) . Also subdividing the pendent vertices connected by x_4 to p_i and x_2 to q_j where $= 1, 2, 3, \dots, m$ and $j = 1, 2, 3, \dots, n$.

The new vertex set formed is $((J(m, n))) = \{x_1, x_2, x_3, x_4 \cup c_1, c_2, c_3, c_4, c_5 \cup p_i \cup u_i \cup q_i \cup v_j\}$. The new edge set formed is $E(((J(m, n)))) = \{(x_1, c_1), (c_1, x_2), (x_2, c_2), (x_3, c_3), (c_3, x_4), (x_4, c_4), (c_4, x_1), (x_1, c_5), (c_5, x_3) \cup (x_4, p_i) \cup (p_i, u_i) \cup (x_2, q_j) \cup (q_j, v_j)\}$, where $i = 1, 2, 3, \dots, n$ and $j = 1, 2, 3, \dots, m$.

Define the mapping $\varphi: V(C(J(m, n))) \rightarrow \{1, 2, 3\}$ such that $\varphi(x_1) = 1, \varphi(x_2) = 2, \varphi(x_3) = 3$.

The remaining vertices can be assigned any one of these colors with the proper dom-coloring. Clearly it

is a proper dom-coloring on vertices and hence we have $\gamma_{d-c}(C(J(m, n))) \geq 3$.

Suppose assume that $\varphi((m, n)) = 4$ by some optimal coloring μ . Then the coloring μ assigns different colors to higher degree non adjacent vertices. Therefore colors on x_1, x_2, x_3 must be distinct.

Now, the fourth color must appear on any of the remaining vertices. If this happens, then there exists any

one pair of color which does not have edge between them which is a contradiction to our assumption. So

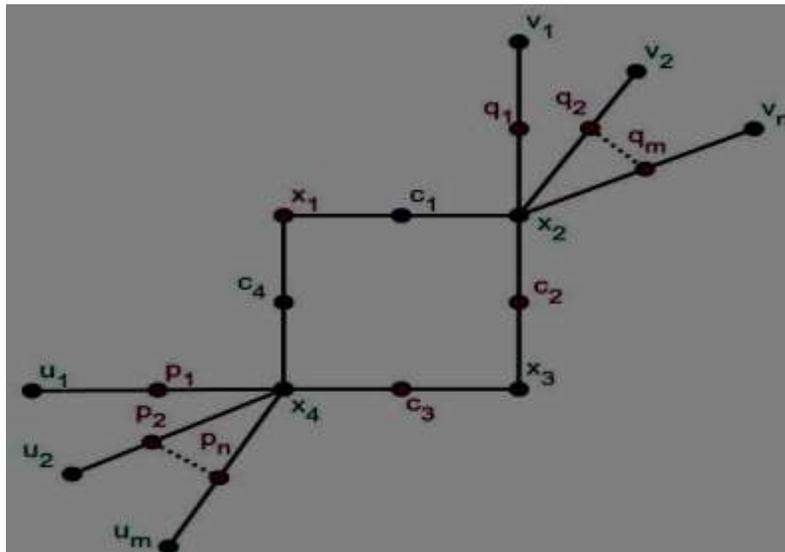
we have $\psi(C(J(m, n))) \leq 3$.

Hence from the inequalities we arrive the required result.

Hence the chromatic number for central graph of jelly fish graph is 3. (ie) $\gamma_{d-c}(C(J(m, n))) = 3$.

We have $\gamma_{d-c}((m, n)) = \chi(C(J(m, n))) + \gamma(C(J(m, n))) - 1$

Therefore $\{d-c\}((J(m, n))) = 3 + 2 - 1 = 4$.



Theorem 3:

Every tree with atleast 2 vertices has dom-chromatic number 2. (ie) $\gamma_{d-c}(T) = 2$. PROOF:

Let T be a tree with $(T) \geq 2$. Let $v \in (T)$.

Consider T be rooted at v . Let v be colored with color 1. Let the neighbours of v be colored with color 2. Let the neighbour of those vertices be colored with color 1. Continue the process of coloring.

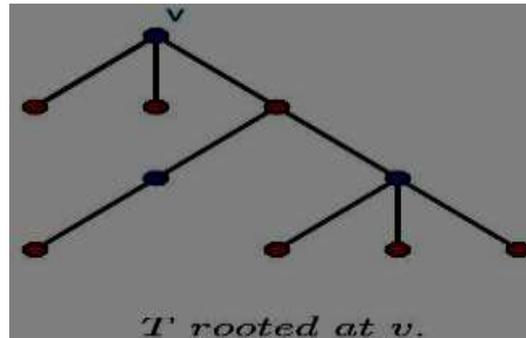
In the tree there is a unique path between any 2 vertices along any path in T . The vertices alternate colors so no pair of adjacent vertices receive the same color. This is 2-colouring.

Therefore, $(T) = 2$.

Therefore, Every tree with atleast 2 vertices is 2-chromatic graph.

We have $\gamma_{d-c}(T) = \chi(T) + \gamma(T) - 1$.

Therefore, $\gamma_{d-c}(G) = 2 + 1 - 1 = 2$

**Theorem 4**

$\gamma d-(G) = 1$ iff G is just a single vertex (K_1).

PROOF:

Assume $\gamma d-(G) = 1$.

To show that G is just a single vertex.

There is one vertex v that dominates the whole graph and v is adjacent to every other vertex, hits every color class in some proper coloring. Then there can be only one color class.

This implies that $(G) = 1$ means no edges, but v is adjacent to every other vertex the only possibility is there must be no other vertices.

Hence G must be exactly K_1 . Conversely,

Assume that G is K_1 .

To prove that $\gamma d-(G) = 1$.

Therefore,

$(G) = 2$.

It is obvious that one vertex dominates itself and hits its only color class.

Therefore, $\gamma d-(G) = 1$

Theorem 5:

For the complete bipartite graph $K_{m,n}$, with $2 \leq m \leq n$, $\gamma d-(K_{m,n}) =$

2. PROOF

The dominating set number is $(K_{m,n}) = 2$, one vertex from each part dominates the whole graph.

The chromatic number is $(K_{m,n}) = 2$, the two parts are the two color classes.

Let S be the set of size of 2 consisting of one vertex from each parts such as S is a dominating set and hits both colors. Thus S is a dominating 2-coloring set, so $\gamma d-(K_{m,n}) \leq 2$. No set of size 1 can be a dom-chromatic set, so $\gamma d-(K_{m,n}) = 2$.

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